intensified and lengthened, and at once began to drift in a direction to the N.W.* Many observers estimated the length of the streak as between 100° and 120° between 8 and 8.30, but this does not include the curious bends at its extremities. At several places the silvery beam or ribbon of light evolved from the meteor was watched for 2 hours: at Plymouth it was distinctly visible for $2\frac{1}{2}$ hours, and another observer 4 miles off Start Point, on the S. Devon coast, followed it for 3 hours. As seen from the south of England, it passed from the stars of Canis Minor and Major, Monoceros and Orion, to Ursa Minor, Cepheus, Cassiopeia, and the constellations lying S.W. It widened from about $\frac{1}{4}°$ to 2° or 3° at last.

The streak was so bright and so extensive that it attracted the earnest attention of thousands of persons who did not see the flight of the meteor at all. This flight was directed from a point in Coma Berenices (190° + 20°) lying low down over the E.N.E. horizon at the time. There is a pretty well-known radiant here in the spring months, and it is composed of slow, trained meteors. A large number of observations have been received both of the fireball of February 22 and its long-enduring streak, and it is hoped to give more definite particulars concerning the main features of the phenomenon in a subsequent paper.

Bishopston, Bristol:
1909 *March* 11.

Comparison of Ancient Eclipses of the Sun with Modern Elements of the Moon's Motion. By Simon Newcomb.

The passages in the writings of ancient authors supposed to refer to total eclipses of the Sun have been so fully discussed during the last few years, especially by Cowell and Nevill in the *Monthly Notices*, and quite recently by Mr. Fotheringham, that the subject is fairly well thrashed out so far as the question of interpretation is concerned. Most of the supposed eclipses on which stress was laid by Airy, Hansen, and other older authorities, in testing the lunar tables, have been nearly eliminated from consideration by doubts and inconsistencies of various kinds. The only one of these I need mention is the eclipse of Thales. The questions associated with this eclipse may now be considered as well cleared up. From the corrections which I have applied to the lunar elements, it would appear that the Sun set upon the combatants only a short time, perhaps fifteen minutes, before the total phase commenced. Thus the accuracy of the phraseology used by Herodotus, and in-

* For an hour or more after its first projection the beam or streak was brighter than any part of the Milky Way, and its shape varied under the action of wind currents at different altitudes. One observer at Lymington describes the streak as like "a brushful of starlight from the Milky Way drawn across the sky."

deed the whole story as he narrates it, seemed to be confirmed in a remarkable way. But the eclipse still remains useless for any astronomical purpose.

After eliminating all the material from which no result can at present be reached, there remain only seven eclipses, the five discussed by Mr. Cowell in the *Monthly Notices*, vol. lxv. pp. 861–867, and lxvi. p. 3, the eclipse of Agathocles, and that of -128, November 20, which Mr. Fotheringham has shown is probably that described as total in the Hellespont while the Sun was four-fifths eclipsed at Alexandria. It was pointed out by Celoria that this last eclipse might have been that of Agathocles, but the year -128 is more probable.

The course by which a conclusion can best be reached is to begin with a careful computation from the best modern theory, and take as the basis of discussion a comparison of the results thus reached with the statements of the historians. This method has not been that hitherto adopted. Oppolzer and Ginzel, in their respective *Canonen*, and Mr. Cowell in his work, start with expressions for the lunar elements constructed so as to represent the eclipses, without reference to their compatibility with modern theory and observations. I need only remark on this that I feel it difficult to be sure of any conclusion reached in this way, and the difficulty is not lessened by the fact that the elements used by the three authorities I have named have no feature in common except that all diverge from what we suppose to be the most probable result of modern theory and observation.

The elements of the Moon's mean longitude which I have used are those derived in the general discussion described in my paper in the *Monthly Notices*, vol. lxix. p. 164, of which the modern basis consists of observed occultations up to 1908. Hansen's motion of the perigee for 1800 has been adopted without change, but its secular acceleration is determined from Brown's theory. The same is true of the node, except that its motion has been corrected by $+11''$ per century, as in Oppolzer's eclipse tables.

The corrections to the mean longitude of Hansen's *Tables de la Lune*, as finally adopted, have the following numerical values. The constant corrections for 1800 have not been applied except in the case of the node, because they are without importance for the present purpose.

Mean Long.	$-26''.57\ T - 4''.231\ T^2 - 0''.0058\ T^3$
Long. Perigee	$-0''.966\ T^2 - 0''.0078\ T^3$
Long. Node	$+11''.00\ T - 0''.629\ T^2 - 0''.0013\ T^3$

T, the time, is reckoned from 1800·0 in Julian centuries. The Sun's mean longitude which I use is that of my own tables.

In computing the eclipses I have used Oppolzer's eclipse tables,*

* Syzygien-Tafeln für den Mond . . . . Publication der Astronomischen Gesellschaft, No. xvi., Leipzig, 1881.

applying the corrections necessary to reduce the elements on which they are based to the adopted values. These corrections are as follows :—

$$
\begin{aligned}
10^4\Delta T &= 5^{d}\cdot 877\ T + 1^{d}\cdot 0460\ T^2 + 0^{d}\cdot 0011\ T^3 \\
10^4\Delta L' &= -\ 2^{\circ}\cdot 17\ \ ,, - 0^{\circ}\cdot 050\ \ ,, \\
10^4\Delta g &= -73^{\circ}\cdot 80\ \ ,, - 13^{\circ}\cdot 79\ \ ,, \\
10^4\Delta \omega &= 0\cdot 00\ \ ,, + 1^{\circ}\cdot 50\ \ ,, - 0^{\circ}\cdot 021\ \ ,, \\
10^4\Delta g' &= -59^{\circ}\cdot 3\ \ ,, + 0^{\circ}\cdot 61\ \ ,,
\end{aligned}
$$

These corrections of L' g, g,′ and ω are for any fixed time. To reduce them to the values for mean conjunction, which form the arguments of the tables, we must apply their motions during ΔT.

The actual corrections to Oppolzer's arguments then become—

$$
\begin{aligned}
10^4\Delta P = 10^4\Delta Q &= +\ 3^{\circ}\cdot 95\ T + 1^{\circ}\cdot 55\ T^2 - 0^{\circ}\cdot 004\ T^3 \\
10^4\Delta I &= +\ 3^{\circ}\cdot 31\ ,, - 0^{\circ}\cdot 13\ ,, + 0^{\circ}\cdot 019\ ,, \\
10^4\Delta II &= -\ 50^{\circ}\cdot 6\ ,, + 0^{\circ}\cdot 47\ ,, + 0^{\circ}\cdot 001\ ,, \\
10^4\Delta III &= +\ 8^{\circ}\cdot 7\ ,, + 3^{\circ}\cdot 5\ ,, \\
10^4\Delta IV &= +53^{\circ}\cdot 9\ ,, - 0^{\circ}\cdot 6\ ,, \\
10^4\Delta V &= -47^{\circ}\cdot 3\ ,, + 0^{\circ}\cdot 3\ ,, \\
10^4\Delta VII &= +57^{\circ}\cdot 2\ ,, - 0^{\circ}\cdot 7\ ,, \\
10^4\Delta VIII &= -44^{\circ}\cdot 0\ ,, + 0^{\circ}\cdot 2\ ,,
\end{aligned}
$$

These corrections to Args. I to VIII are expressed in centesimal degrees, as in Oppolzer's tables. That of Arg. VI is unimportant. All are applicable to the body of Oppolzer's tables, and replace his empirical corrections given in the first four pages of his tables.

The method of computing the eclipses has been to take as fundamental elements the vertical ecliptical ordinate of the shadow-axis at the moment of true conjunction in longitude, together with the hourly motion of each coordinate. The results are then transformed at once to equatorial coordinates x and y, expressed in the Besselian form. But the zero of time is first transferred from T to some moment near the time of greatest phase. The coordinates of the point of observation, ξ and η, and their hourly motions, ξ' and η', being also computed, the nearest approach of the shadow-axis to the point of observation is readily found. It will be seen that I have ignored the geographical position of the shadow-path, which is not required for our purpose.

The next step is to express the increment of minimum distance in terms of corrections to the elements. The formulæ of computation are developed in my *Researches on the Motion of the Moon*. For convenience of verification I cite them.

T being the G.M.T. of true conjunction in longitude, we choose a moment $T + t$, expressed in hours, near that of greatest phase at

the place. For this time we find the local West hour-angle of the shadow-axis

$$\mu' = (T + t) \times 15^\circ - Z + \lambda$$

Z being the equation of time, and λ the East longitude of the place.

The coordinates ξ and η and their hourly motions are then found from

$$\xi = \rho \cos \phi' \sin \mu'$$
$$\eta = \rho \sin \phi' \cos d - \rho \cos \phi' \sin d \cos \mu'$$
$$\xi' = [9{\cdot}4192] \rho \cos \phi' \cos \mu'$$
$$\eta' = [9{\cdot}4192] \xi \sin d$$

where d is the declination of the shadow-axis, for which we may take that of the Sun.

For the Besselian (equatorial) coordinates of the shadow-axis, on the fundamental plane we need the Sun's Dec. and the angle h which its motion in longitude makes with the parallel of declination. For these we have, putting ϵ for the obliquity,

$$\cos d \sin h = \sin \epsilon \cos L'$$
$$\cos d \cos h = \cos \epsilon$$
$$\sin d \quad = \sin \epsilon \sin L'$$

Here we may correct L′ for the motion during t, but the effect of the correction will be small.

The ecliptical coordinates of the shadow-axis at the time T in terms of the Oppolzer elements are—

$$x_2^{(0)} = 0\ ;\ y_2^{(0)} = p \sin P$$

The hourly motions of these coordinates, which we regard as constant, are—

$$x'_2 = \Delta L\ ;\ y'_2 = q \cos Q$$

Their values at the time $T + t$ are

$$x_2 = x'_2 t\ ;\ y_2 = y_2^{(0)} + y'_2 t$$

The equatorial coordinates are

$$x = x_2 \cos h - y_2 \sin h$$
$$y = x_2 \sin h + y_2 \cos h$$

which may be computed with the values of x_2 and y_2 for the moment $T + t$. A familiar method of computing x, y, and their hourly motions is

$$m \sin M = x_2\ ; \qquad n \sin N = x'_2$$
$$m \cos M = y_2\ ; \qquad n \cos N = y'_2$$
$$x = m \sin (M - p)\ ;\ x' = n \sin (N - p)$$
$$y = m \cos (M - p)\ ;\ y' = n \cos (N - p)$$

We now find Δ, the minimum distance of the point of observation from the shadow-axis, by the equations

$$X = x - \xi \qquad X' = x' - \xi'$$
$$Y = y - \eta \qquad Y' = y' - \eta'$$
$$\Delta = \frac{X'Y - XY'}{\sqrt{X'^2 + Y'^2}}$$

Finally, we have to know the effect of a change in the elements upon Δ. The only elements as to which there can be any question are the longitudes of the Sun, the Moon and the node. If we put

v, β, π, the moon's longitude, latitude, and parallax;
L' the Sun's longitude;

we compute S from

$$\tan S = \frac{Y'}{X'} \quad . \quad . \quad . \quad . \quad (90^\circ > S > -90^\circ)$$

Then

$$\frac{\delta\Delta}{\partial v} = -\frac{\delta\Delta}{\partial L'} = \frac{\sin(p - S)}{\sin \pi}$$
$$\frac{\delta\Delta}{\partial\beta} = \frac{\cos(p - S)}{\sin \pi}$$

We have to replace v and β by λ, the Moon's mean longitude, and ☊, that of the node. We have, with all necessary precision, when the Moon is near conjunction,

$$\frac{\delta v}{\partial\lambda} = 1{\cdot}019 + 0{\cdot}131 \cos g \equiv k$$

where

$$g = \text{Arg. I} \times 0^\circ{\cdot}9$$

Then

$$\frac{\delta\Delta}{\partial\lambda} = k\left(\frac{\delta\Delta}{\partial V} \pm 0{\cdot}0895\frac{\delta\Delta}{\partial\beta}\right)$$
$$\frac{\delta\Delta}{\partial ☊} = \mp 0{\cdot}0895\frac{\delta\Delta}{\partial\beta}$$

The upper sign is used when the eclipse is near the ascending node, and the lower one near the descending node.

The results of the whole computation are tabulated as follows. T, L′, etc., are the elements from my theory in Oppolzer's notation. Δ is given in units of the 4th. place of decimals of the Earth's equatorial radius. Each Δ is followed by the increment which it would receive from an increment of λ, the Moon's mean longitude at the time, of ☊, the longitude of the node, and of L′ the Sun's true longitude, all, for convenience, expressed in minutes of arc.

Elements of Seven Ancient Eclipses.

T.		L′.	Z.	P.	Q.	log p.	log q.	log ΔL.
− 1062 July	30·7548	116°·332	+0°·11	177°·858	175°·90	0·7005	8·7490	9·7528
− 762 June	4·8691	74·585	−2·08	176·547	174·74	0·6988	8·7509	9·7555
− 647 April	5·8765	8·925	+0·75	171·597	172·14	0·6909	8·7594	9·7638
− 430 Aug.	3·1327	124·487	+0·52	170·577	172·95	0·7233	8·7262	9·7277
− 309 Aug.	14·8505	136·573	+0·52	3·736	2·66	0·6929	8·7574	9·7612
− 128 Nov.	20·0565	236·080	−2·79	9·403	11·40	0·6998	8·7490	9·7509
+ 197 June	2·9958	71·068	−1·61	1·702	4·03	0·7245	8·7249	9·7268

Node.		Date.	Minimum Distance of Shadow-axis.				Radius of Shadow.
☋	− 1062	July 30	Δ = +516 − 16·2Δλ	+ 15·0☊	− 0·3ΔL′		+ 94
☋	− 762	June 15	,, +684 − 3·3	+ 14·8	− 11·9		+ 151
☋	− 647	April 6	,, +567 − 25·7	+ 14·6	+ 7·7		+ 178
☋	− 430	Aug. 3	,, +577 + 23·4	+ 15·3	− 38·7		− 55
☊	− 309	Aug. 14	,, −227 − 7·9	− 14·6	+ 21·5		+ 145
☊	− 128	Nov. 20	,, +018 − 20·9	− 14·7	+ 33·6		+ 15
☊	+ 197	June 3	,, −436 − 14·3	− 15·6	+ 30·0		− 6

The preceding eclipses, that of − 128 excepted, were discussed by Mr. Cowell. He shows that five of the eclipses in question can all be represented in two ways, either by a change in the secular acceleration of the node, or by a hitherto unsuspected acceleration of the Sun's longitude, combined with an equal correction to the Moon's secular acceleration.

It will be seen that the preceding equations are confirmatory of Mr. Cowell's results. A diminution of 36′ in the longitude of the node will make all five of the eclipses nearly or quite annular or total as the case may be. This effect will be brought about by a diminution of about 3″·5 in the secular acceleration of the node.

In the equations it will be seen that the sums of the coefficients of the increments are nearly evanescent. It follows that the representation of the eclipses will remain nearly unchanged when we assign arbitrary equal increments to the longitudes of the Moon, the Sun, and the node. Hence, instead of a diminution of 3″·5 in the nodal acceleration, we may assign an increment of 3″·5 to the accelerations of the Sun and Moon. All this is quite accordant with Cowell's conclusions.

On the other hand, Cowell's corrections make both the eclipse of − 309 and that of − 128 only partial in the Hellespont. We have therefore to set against the evidence of the five the contrary evidence afforded by the eclipse of − 128 that the elements admit of no correction.

It may also be remarked that an increase of 1″ in the secular acceleration of the mean longitude, coupled with a diminution of 1″ in that of the node, would suffice to make the eclipses of − 1062 and − 647 total, without throwing that of − 128 quite away from

the Hellespont. But this change in $\Delta^2 \Omega$ is outside the limit of theoretical uncertainty, and in $\Delta^2\lambda$ is difficult to admit.

The question whether the remarkable coincidence among the five eclipses affords probable evidence that either set of corrections to the elements is real is one on which opinions will doubtless differ. It must certainly be admitted that either correction seems extremely improbable. In the case of the node, gravitational theory admits of no appreciable correction to the adopted secular acceleration. That any other cause than gravitation changes the motion of the node is rendered improbable by the close agreement between the actual motion and the theoretical motion derived by Brown.

Equally difficult of explanation on any theory is a secular acceleration of the Earth's orbital motion. Moreover, while modern observations, so far as I have discussed them, do not exclude the possibility of such an acceleration, they do render it improbable.

The other branch of the question is, whether a chance coincidence in five cases out of six can be regarded as conclusive. The evidence in favour of the actual centrality at the several places of record of the eclipses, as the accounts are cited by Mr. Cowell, seems to me rather weak. It is certain that the eclipses were central within a few hundred miles of the several points, but, in the absence of any specific statement of the place, I see nothing stronger than a greater or less presumption of centrality at the supposed places.

The eclipse of the list in which the question of the phase is most perplexing is that of -430, described by Thucydides, and also mentioned by Cicero and Valerius Maximus, as seen at Athens. Both of these accounts are weak from proving too much, namely, a total eclipse. If there could be any question whether the eclipse was annular or total, the combined weight of the two accounts would lead us to regard totality at Athens as practically certain, because no other phase could have been one of darkness, caused alarm among the citizens, or made the stars visible. We must therefore make a large allowance for exaggeration; and the question is whether this allowance can be much smaller for an annular eclipse than for the partial eclipse which is computed from the tables. The most exact basis for a comparison is afforded by comparing the fraction of the Sun's disk which must have remained uncovered in the two hypothetical cases. I find that if the eclipse was annular at Athens, the breadth of the annulus was about 18″. If it were partial, a rough computation shows that the area of the Sun's disk uncovered was about four times that of the annulus—possibly nearly five times. At first sight this difference might seem important, but a little consideration may lead us to minimise this importance. The difference in the intensity of daylight under a cloudy sky may well be much greater than this without being striking. The light of the sky a very few minutes after sunset is less than it was during either of the hypothetical phases. Those who have observed total eclipses of the Sun know how little

striking is the apparent diminution in the intensity of daylight up to within a few minutes of the total phase, and how suddenly the first ray of the reappearing Sun illumines the sky and blots out the stars. Computation shows that Venus was very favourably situated for observation during the eclipse of −430, and it may well be doubted whether any other star or planet could have been visible, even if the phase were annular. It is also to be considered that the ratio of the illuminations of the sky in the two cases is less than that of the uncovered Sun. Altogether, we may make an ample allowance for exaggeration and misapprehension without overstepping the bounds of reasonable probability.

Note on Professor Newcomb's Paper on "Comparison of the Ancient Eclipses of the Sun with Modern Elements of the Moon's Motion." By J. K. Fotheringham, M.A.

(*Communicated by Professor H. H. Turner, D.Sc., F.R.S.*)

[This note was written before the paragraph was added to Professor Newcomb's paper: "It may also be remarked that an increase of 1″ in the secular acceleration of the mean longitude . . . . is difficult to admit."]

Of the eclipses discussed in Professor Newcomb's paper, two only, those of −1062 and −128, are definitely described as total, within a closely defined locality. It has therefore occurred to me to seek values which will satisfy these two eclipses. I have to thank Professor Turner for kind assistance in the following discussion.

Professor Newcomb's equations for these eclipses are

$$-1062 \qquad -16\Delta\lambda + 15\Delta\Omega - 0\Delta L' + 516 = 0.$$

$$-128 \qquad -21\Delta\lambda - 15\Delta\Omega + 34\Delta L' + 018 = 0$$

These give practically

$$\Delta(\lambda - \Omega) = \frac{516}{16} = 32.$$

$$\Delta(L' - \Omega) = \frac{21 \times 32 - 18}{34} = \frac{654}{34} = 19.$$

But this ignores the fact that these quantities increase as t^2. If we adopt their values for the second eclipse as standard (roughly 2000 years ago), the first eclipse (which was 3000 years ago) must